

# “Pipetron” Beam Dynamics with Noise

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## **Abstract**

Extra-large hadron collider – “Pipetron” – at 100 TeV energy range is currently under consideration. In this article we study the Pipetron transverse and longitudinal beam dynamics under influence of external noises. The major effects are growths of transverse and longitudinal emittances of the beam caused by noisy forces which vary over the revolution period or synchrotron oscillation period, respectively; and closed orbit distortions induced by slow drift of magnet positions. Based on analytical consideration of these phenomena, we estimate tolerable levels of these noises and compare them with available experimental data. Although it is concluded that transverse and, probably, longitudinal feedback systems are necessary for the emittances preservation, and sophisticated beam-based orbit correction methods should be used at the Pipetron, we observe no unreasonable requirements which present an impenetrable barrier to the project.

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# 1 Introduction

Several proposals of the post-LHC large colliders with 30–100 TeV beam energy and  $10^{33} - 10^{35} \text{ s}^{-1} \text{ cm}^{-2}$  have been considered in recent years. Two approaches can be distinguished in the trend – namely, smaller circumference ring with high magnetic field dipoles based on high- $T_c$  technology [1], and presumably lower cost option of a micro-tunnel low-field machine with consequently large circumference [2]. The later – often referred as “Pipetron” (or “MegaCollider”) – is a subject of this article. Table 1 shows relevant parameters of the collider [3].

Table 1: “Pipetron”- MegaCollider parameters

Proton Energy,	$E_p$ , TeV	100
Circumference,	C, km	1000
Luminosity,	$L$ , $\text{s}^{-1} \text{ cm}^{-2}$	$10^{35}$
Intensity,	$N_p/\text{bunch}$	$4.1 \cdot 10^{10}$
No. of Bunches,	$N_b$	25000
RMS emittance,	$\epsilon_n$ , $10^{-6} \text{ m}$	1
Long. emittance (rms),	$A$ , eV·sec	0.3
Bunch length (rms),	$\sigma_s$ , cm	10
Mom.spread (rms),	$\Delta P/P$	$10^{-5}$
Rev. frequency,	$f_0$ , Hz	300
Interaction focus	$\beta^*$ , cm	10
IP size	$\sigma_{IP}$ , $\mu\text{m}$	1
Beam-beam tune shift	$\xi_p$	0.005

The collider ring consists of thousands of magnetic elements, and their field imperfections can seriously affect proper machine operation. It is known [5] that depending on the frequency band one can distinguish two mechanisms of beam perturbations in circular accelerator. Slow processes (with respect to revolution period) produce a distortion of the closed orbit of the beam. At higher frequencies (comparable with the revolution frequency), noises cause direct emittance growth. The revolution frequency of the Pipetron is much lower than in any other existing or ever planned accelerator, so, because numerous natural noises rapidly grow with frequency decrease, the noise may produce dramatic effect on the beam dynamics

of the Pipetron. This article is devoted to major effects in beam dynamics due to external noise. Besides this Introduction, the paper consists of four chapters devoted to transverse emittance growth, longitudinal emittance growth, closed orbit drifts, and comparison of the Pipetron tolerances with those of the LHC and the SSC. The final chapter summarizes major conclusions.

## 2 Transverse Emittance Growth

### 2.1 Effect of Transverse Kicks

**Transverse kicks.** The primary sources which lead to emittance growth in large hadron colliders are quadrupoles (quad) jitter and high-frequency variations of the bending magnetic field in dipoles. Both sources produce angular kicks and excite coherent betatron oscillations. After some time (which is about 1200 turns in the case of the Pipetron – see below in the section devoted to a feedback system) filamentation or dilution process due to tune spread within the beam transforms the coherent oscillations into the emittance increase. If there is no damping of the excited coherent motion, then the latter as whole “smears” to the beam phase space volume. In the simplest case, when the kick amplitude  $\Delta\theta$  varies randomly after the revolution time  $1/f_0$  and its variance is  $\delta\theta^2$ , one can estimate the transverse emittance growth as:

$$\frac{d\epsilon_n}{dt} = \frac{1}{2}f_0\gamma \sum_i^{all\ kicks} \Delta\theta_i^2\beta_i = \frac{1}{2}f_0\gamma\delta\theta^2 \langle \beta \rangle N \quad (1)$$

where  $\langle \beta \rangle$  is the average beta function,  $\gamma = E_p/mc^2$  is relativistic factor, and  $N$  is the number of elements which produce uncorrelated kicks. Two major sources of the dipole kicks are fluctuations  $\delta B$  of the bending dipole magnetic field  $B_0$  which give horizontal kick of  $\delta\theta = \theta_0(\delta B/B_0)$  ( $\theta_0 = 2\pi/N_d$  is bending angle in each dipole,  $N_d$  is total number of dipoles); and transverse quadrupole magnets displacements  $\delta X$  which lead to kick of  $\delta\theta = \delta X/F$ , where  $F$  is the quadrupole focusing length. For a ring which consists mostly of FODO focusing structure with half cell length of  $L$  (approximately equal to dipole magnet length) and the phase advance per cell of  $\mu$  one can rewrite the emittance growth rate equation <sup>1</sup>:

$$\frac{d\epsilon_n}{dt} = 2f_0\gamma \frac{\delta X^2 N_q}{L} tg(\mu/2) = 2c\gamma \frac{\delta X^2}{L^2} tg(\mu/2) = \gamma \frac{\delta X^2}{c} N_q^2 2tg(\mu/2), \quad (2)$$

where  $N_q$  is total number of quads,  $c$  is the speed of light. Similarly, uncorrelated field fluctuations in dipoles result into mostly horizontal emittance growth rate – while (2) stands for both vertical and horizontal emittances – equal to:

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<sup>1</sup>following Ref. [4], we take into account FODO equation  $\sum_i \beta_i/F_i^2 = 4tg(\mu/2)N_q/L$

$$\frac{d\epsilon_n}{dt} = \frac{\pi f_0 \gamma L}{\nu} \frac{\delta B^2}{B_0^2} = \frac{\pi c \gamma}{N_d \nu} \frac{\delta B^2}{B_0^2}, \quad (3)$$

$\nu = C/(2\pi\nu)$  is the tune.

It is interesting to note, that “vibrational” emittance growth (2) is proportional to factor of  $N_q^2 t g(\mu/2) \propto N_q \nu = \Phi$ , while dipole field effect (3) is proportional to  $\Phi^{-1}$ . The value of  $\Phi$  is proportional to  $\nu$  if the half-cell length value  $L$  is fixed, or grows as  $\nu^2$  if the phase advance per cell  $\mu$  is constant. Therefore, the two contributions to the emittance growth rate (2,3) perform exactly opposite dependencies on the machine tune.

In general case, when external noise is not “white” (exactly random in time) and can be described by power spectral density  $S_{\delta\theta}(f)$ <sup>2</sup> which depends on frequency  $f$ , the emittance growth rate is calculated in [5]:

$$\frac{d\epsilon_n}{dt} = \gamma f_0^2 \sum_i \left( \beta_i \text{Sum}_i(\nu) \right) \quad (4)$$

where

$$\text{Sum}_i(\nu) = \sum_{n=-\infty}^{\infty} S_{\delta\theta}(f_0|\nu - n|) \quad (5)$$

is the sum of power spectral densities of angular kicks produced by the  $i$ -th source at frequencies of  $f_0|\nu - n|$ ,  $n$  is integer, the lowest of them is fractional part of the tune times revolution frequency  $f_1 = \Delta\nu f_0$  ( $\beta_i$  is the beta function at the  $i$ -th magnet). The dimension of  $\text{Sum}(f)$  is 1/Hz, so the dimension of the emittance growth rate is meters/sec. Note, that we assume that kick sources are uncorrelated.

**Beam lifetime and acceptable emittance growth.** Let us constrain that external noise should lead to less than 10% emittance increase while the beam circulates in the accelerator. Characteristic beam lifetime  $\tau$  in Pipetron has to be chosen to optimize integrated luminosity. Several time constants play role in that. First of all, these are longitudinal and transverse *emittance* growth times due to intrabeam scattering, which are equal to (see, e.g. [6]):

$$\tau_{\parallel}^{IBS} \approx \frac{4\epsilon_n^2 A \nu_x d}{\pi L_c m_p c^2 N_p r_p^2}, \quad d^2 = 1/(1 + \frac{\sigma_x^2}{D_x^2 (\Delta P/P)^2}) \approx \frac{c A \nu_s}{\epsilon_x E_p \nu_x}, \quad (6)$$

and

$$\tau_x^{IBS} \approx \tau_{\parallel}^{IBS} / d^2, \quad L_c = \ln \frac{\gamma^{1/4} \nu_x^{1/4} \epsilon_x^{3/4}}{R^{1/4} r_p^{1/2}} \quad (7)$$

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<sup>2</sup>see definitions of the power spectral density in the next section concerning ground vibration noise

where  $r_p = 1.53 \cdot 10^{-18} \text{ m}$  is proton's classical radius,  $R = C/2\pi$  is the ring radius, and  $\nu_x$  is the horizontal betatron tune. Taking for definiteness  $\nu_x \approx 500$  (see below) one gets  $\tau_{\parallel}^{IBS} \approx 6 \text{ hrs}$ , and  $\tau_x^{IBS} \approx 500 \text{ hrs}$ . The luminosity “burn-up” time  $\tau_L = N_p N_b / (L \sigma_{pp}) \approx 28 \text{ hours}$  ( $\sigma_{pp} \approx 100 \text{ mb}$  is total  $pp$  cross section at 100 TeV). Transverse damping time  $\tau_D$  due to synchrotron radiation of protons in Pipetron is about 42 hours, that is too small for the radiation to play any significant role in beam dynamics.

Comparing these temporal values one can choose the Pipetron cycle time of about  $\tau_c = 5 \text{ hours}$  and get the constraint on the noise-induced emittance growth:

$$\frac{d\epsilon_n}{dt} \leq 0.1 \frac{\epsilon_n}{\tau_c} = 5.6 \cdot 10^{-12} \text{ m/s}. \quad (8)$$

**Tolerances.** Taking into consideration 500-m long FODO cell (i.e.  $L = 250 \text{ m}$ ) focusing structure with  $\mu = 90^\circ$  phase advance per cell [3] one can estimate the tune  $\nu \simeq 500$ , total number of focusing quadrupoles as  $N_q = 4000$  and about the same number of dipoles  $N_d$ . Now, the acceptable transverse emittance growth rate requires:

- single quadrupole transverse vibration spectral density of power is limited by the value of:

$$\sum_n S_{\delta X}(f_0|\nu - n|) \approx S_{\delta X}(f_0 \Delta \nu) \leq 2 \cdot 10^{-11} \frac{\mu\text{m}^2}{\text{Hz}} = 20 \frac{\text{pm}^2}{\text{Hz}},$$

where  $\Delta \nu$  is fractional part of  $\nu$ . Approximation sign reflects that spectrum of vibrations falls fast with frequency increase (see below).

- or the rms amplitude of turn-to-turn jitter of each quadrupole (white noise in frequency band  $f_0^3$ ):

$$\delta X_{rms} \leq 0.76 \cdot 10^{-10} \text{ m} = 0.76 \cdot 10^{-4} \mu\text{m} = 0.76 \text{ \AA}.$$

- and a tolerable level of bending magnetic field fluctuations to its mean value  $B_0$  in the dipole:

$$\left( \delta B / B_0 \right)_{rms} \leq 3.4 \cdot 10^{-10}.$$

## 2.2 Measured Ground Motion

Let us make a comparison of the above calculated constraints with experimental data. First of all, one should consider the ground motion because it is ambient, always existing and non-controlled noise. Technological near-by equipment can increase

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<sup>3</sup>note, that transition between “white noise” formula (1) to “color noise” one (5) corresponds to substitution  $\delta X^2 \Leftrightarrow f_0 S_X(\Delta \nu f_0)$

natural vibrations level by several orders of magnitude. In addition, accelerator environment contains many other sources which can produce angular kicks and, therefore, initiate the emittance growth (see, e.g. Tevatron experience in [23]). In recent years a number of thorough experimental investigations of ground vibrations have been done for future colliders (see review in [7]). Below we outline some results.

As most of disturbances are *noises*, then statistical spectral analysis defines the *power spectral density*  $S_x(f)$  (PSD) of noise process  $x(t)$  at frequency  $f \geq 0$  as:

$$S_x(f) = \lim_{T \rightarrow \infty} \frac{2}{T} \left| \int_0^T x(t) e^{-i2\pi f t} dt \right|^2. \quad (9)$$

The dimension of the PSD is *power in unit frequency band*, e.g.  $m^2/Hz$  for the PSD of displacement. PSD relates to the *rms value of signal*  $\sigma_{rms}(f_1, f_2)$  in the frequency band from  $f_1$  to  $f_2$  as  $\sigma_{rms}^2(f_1, f_2) = \int_{f_1}^{f_2} S_x(f) df$ , e.g. below we note integrated rms amplitude that corresponds to  $f_2 = \infty$ . The *spectrum of coherence*  $C(f)$  of two signals  $x(t), y(t)$  is defined as:

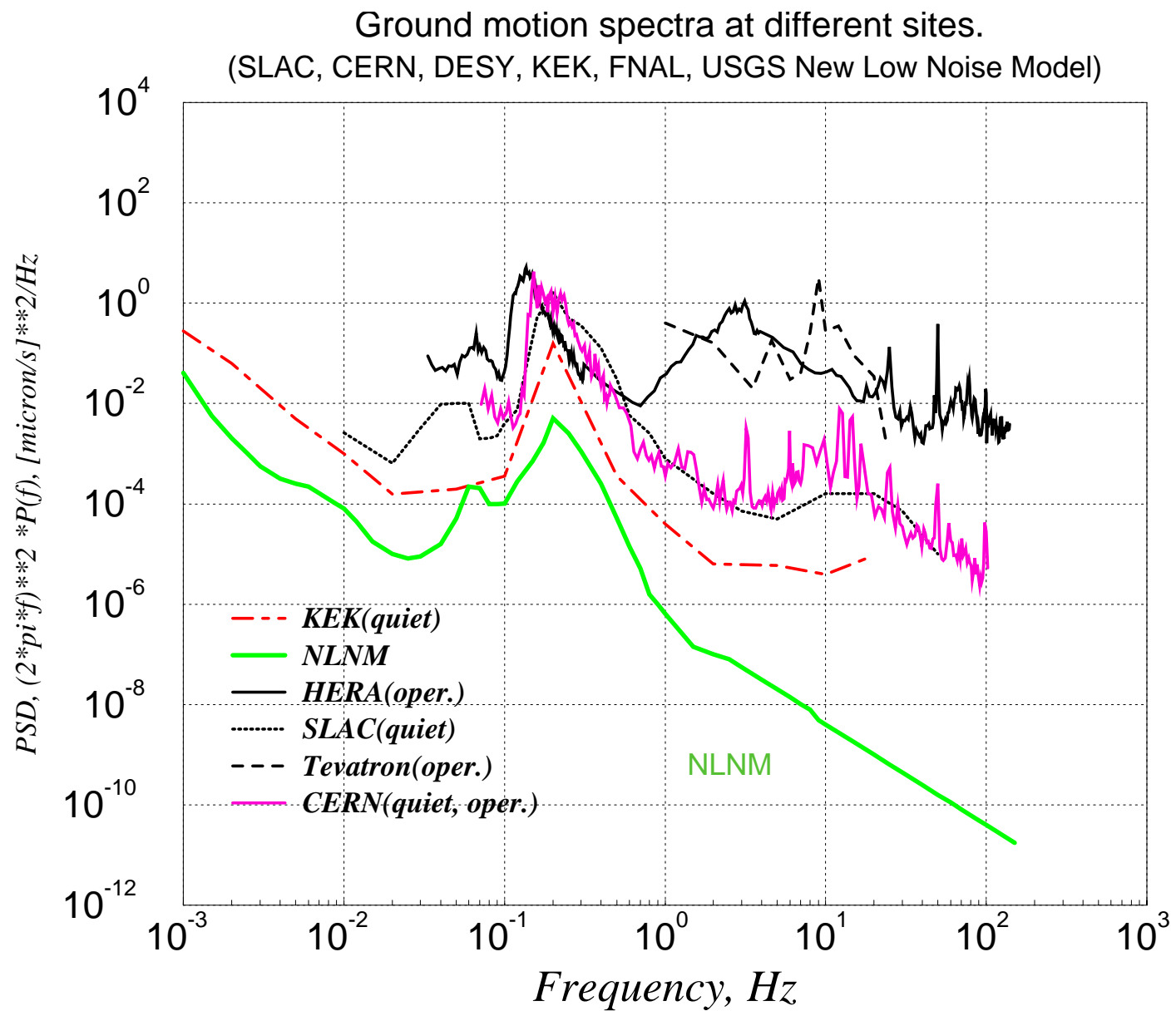
$$C(f) = \left| \frac{\langle X(f)Y^*(f) \rangle}{\sqrt{\langle X(f)X^*(f) \rangle \langle Y(f)Y^*(f) \rangle}} \right|, \quad (10)$$

here  $\langle \dots \rangle$  means averaging over different measurements and  $X(f), Y(f)$  are Fourier transformations of  $x, y$ . The coherence does not exceed 1.0 and is equal to 0 for completely uncorrelated signals.

Fig.1 compares the value of  $S_x(f)(2\pi f)^2$  in units of  $(\mu m/s)^2/Hz^4$  for the US Geological Survey “New Low Noise Model” [8] – a minimum of the PSD observed by geophysicists worldwide – and data from accelerator facilities of HERA [9], KEK [10], CERN [12], SLAC[14], and FNAL [15]. These PSDs of velocity indicate that: 1) accelerators are essentially “noisy” places; 2) ground vibrations above 1 Hz are strongly determined by cultural noises – they manifest themselves as numerous peaks in Fig.1; 3) even among accelerator sites the difference is very large, that gives a hint for the Pipetron builders.

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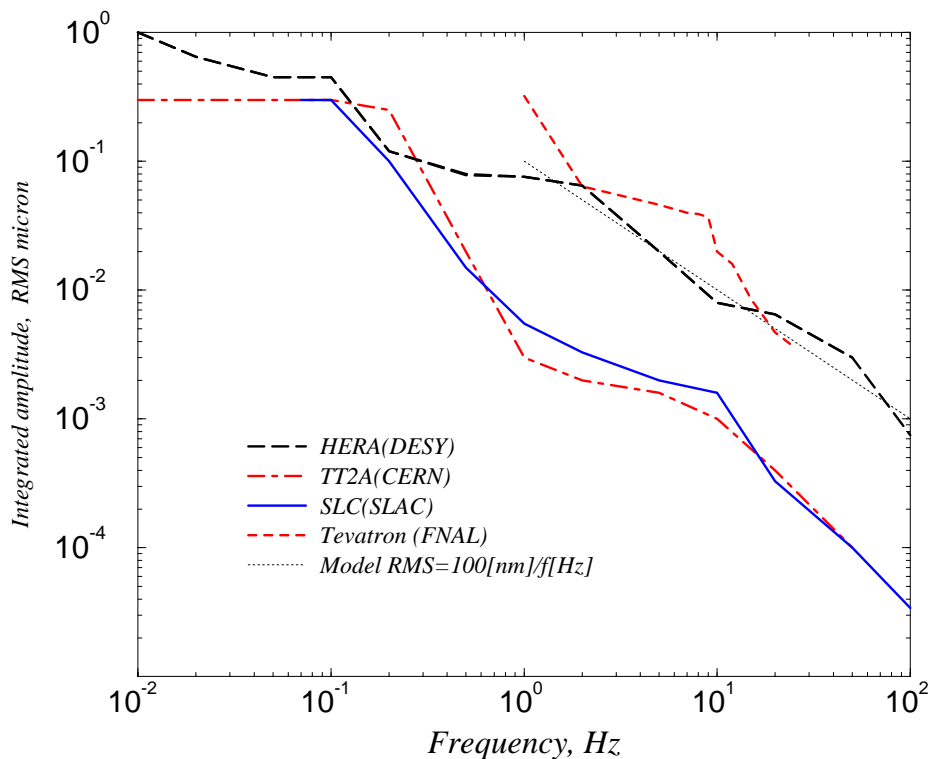
<sup>4</sup>i.e. the PSD of velocity  $v = 2\pi f x$ . The ground velocity spectra plots are looking much better than the PSDs of displacement  $x$  which look very tilted because of strong reduction of noises at higher frequencies.





There is a “rule of thumb” [7] that says that the rms amplitude of the vibration at frequency  $f$  and above is equal to  $r.m.s. X = B/f[Hz]$  (here  $B$  is a constant) which corresponds to the PSD of  $S_x(f) = 2B^2/f^3$ . Within a factor of 4 this rule usually fits well the accelerators-averaged vibration amplitudes above 1 Hz under “quiet” conditions. Fig.2 presents the values of  $rms X(f) = \int_f^\infty S_x(f)df$  calculated for several spectra from Fig.1 – namely, for SLAC, CERN, HERA, and FNAL data. The measurement of tunnel floor vibration amplitude made in the Tevatron tunnel at FNAL covers frequencies of 1–25 Hz and can be approximated by the “rule of thumb” with  $B = 100$  nm. Although there is no data on FNAL site vibrations at higher frequencies, we will use the fit predictions above 25 Hz as well. From Fig.2 one can see that almost the same coefficient  $B$  is applicable for the HERA tunnel amplitudes, while ground motion amplitudes in tunnels of SLC(SLAC) and TT2A(CERN) are about 10-20 times smaller.

Below 1 Hz the ground motion amplitude is about 0.3-1  $\mu\text{m}$  due to remarkable phenomena of “7-second hum”. This hum is waves produced by oceans – see a broad peak around 0.14 Hz in Fig.1 – with wavelength of about  $\lambda \simeq 30$  km. It produces negligible effect on Pipetron, because  $\lambda$  is much bigger than typical betatron wavelength  $2\pi\beta \simeq 2$  km.



**Figure 2:** RMS amplitude above  $f$  vs.  $f$ .

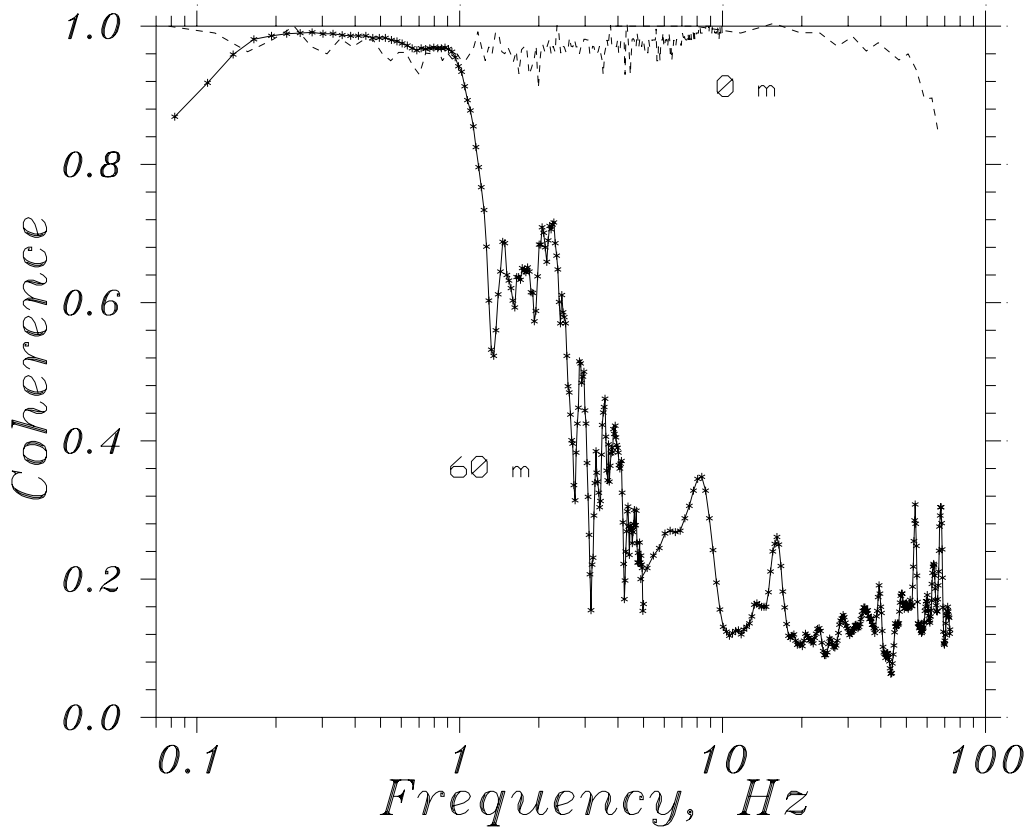
Thorough investigations of spatial characteristics of the fast ground motion have shown that above 1-4 Hz the correlation significantly drops at dozens of meters of distance between points. Fig.3 shows the spectrum of coherence between vibrations of two quadrupoles distanced by 60m at the APS(ANL) [13]. The coherence falls with increasing distance  $L$  between observation points, and sometimes a 2-D random waves model prediction of  $C(f) = |J_0(2\pi fL/v)|$  with  $v = 200 - 500\text{m/s}$  fits well to the experimental data [14]. For the FODO lattice with distance between quads  $L = 250$  one may treat motion of magnets as uncorrelated at frequencies above 1 Hz.

Table 2 compares requirements for the Pipetron with three particular tunes  $\Delta\nu = 0.18, 0.31$  and  $0.45$  and experimental data. Note that corresponding frequencies  $f_1 = f_0\Delta\nu$  are equal to 54 Hz, 93 Hz, and 135 Hz.

Table 2: PSD of Ground Motion (in  $(pm)^2/Hz$ ) at Three Frequencies

$\Delta\nu$	0.18	0.31	0.45
$f_1$	54 Hz	93 Hz	135 Hz
<b>Pipetron tolerance</b>	<b>20</b>	<b>20</b>	<b>20</b>
NLNM	0.02	$2 \cdot 10^{-3}$	$2 \cdot 10^{-4}$
SLAC (quiet)	100	-	-
DESY (tunnel)	$10^5$	7000	1700
SSC (quiet) [11]	$10^4$	100	20
CERN (tunnel)	300	20	-
<b>“Rule of thumb”</b>	<b><math>1.3 \cdot 10^5</math></b>	<b><math>2.5 \cdot 10^4</math></b>	<b>8000</b>

One can see that none of the accelerator data shows vibrations which are less than the Pipetron requirements, although PSDs at higher frequencies (say  $f_1 = 135$  Hz) are much less than at lower frequency of 54 Hz, and, therefore, larger  $\Delta\nu$  – closer to half integer resonance – are preferable from this point of view. At  $\Delta\nu = 0.18$  one needs the vibration power reduction factor of  $R = 10 - 10^4$ .



**Figure 3:** Coherence spectra at the APS (ANL).

Before discussion on the feedback system which can effectively counteract the emittance growth, we'd like to make three comments: firstly, there are ways to reduce quadrupole vibrations with active mechanical stabilization of the magnets or passive dampers which isolate magnets from sources of vibrations (ground, cryogenic/electrical systems, etc.). The active stabilization of magnetic elements - besides its probable high cost for the really large accelerator - doesn't seem to be applicable for damping at frequencies above 20-30 Hz (see e.g. [16]). In opposite, the passive isolation works better at higher frequencies, although its capability is quite limited (characteristic damping of 10-20dB [17]), but it leads to certain degradation of low-frequency stability and does not cure vibrations produced inside the magnet.

Secondly, requirement on the magnet motions is somewhat easy in the combined function lattice. Indeed, from 1, one can see that if the characteristic length over which mechanical motion of the dipole+quadrupole in one magnet can be considered as coherent is equal to  $l_c$ , than the emittance growth rate is  $r = l_c/L$  times less than (1)<sup>5</sup>. At frequencies about 50-100 Hz and above one can roughly estimate  $l_c \sim 10$

<sup>5</sup>indeed, the number of coherently vibrating sub-quads with length  $l_c$  is proportional to  $N_c \propto 1/r$  while the kick produced by each of them is  $r$  times weaker  $\Delta\theta_c \propto r$ , thus the total effect in the emittance growth is proportional to the product of  $N_c$  and  $\Delta\theta_c^2$  that is  $\propto (1/r) * r^2 = r$ .

m, so, as  $L = 250$  m, we obtain  $r \simeq 1/25$  and, consequently, 5 times larger tolerance on the ground motion *amplitude*. Unfortunately, variations in the PSD of ground motion are at least hundred times larger than  $r$ , thus, the combined function lattice can not solve the whole problem.

Thirdly, we have not enough experimental data to answer the question: “Is it possible to reduce dipole field fluctuations at 50-150 Hz down to the level of  $3 \cdot 10^{-9}$ ?”. At these frequencies the skin depth even in copper is about 1 cm, thus, no reasonable vacuum chamber can effectively reduce field variation due to current ripple. Another important and unanswered question is spatial coherence of the current ripple: correlated field changes over the ring can lead to substantial increase as well as decrease of the emittance growth. To avoid confusion, we should note, that in contrast to a wideband noise, the main components of the ripple are usually concentrated at several well-defined frequencies (multiples and subharmonics of 60 Hz in the USA), and one can significantly reduce their detrimental influence by detuning  $f_1 = \Delta\nu f_0$  away from these frequencies.

## 2.3 Feedback System

**Emittance evolution.** A transverse feedback frequency allows one to suppress the emittance growth caused by excitation of the betatron oscillations by external noise kicks simply by damping the coherent beam motion which otherwise goes directly to the beam phase space increase. It is obvious that the oscillations should be damped much faster then they decohere. The system monitors the dipole offset  $X$  of the beam centroid and tries to correct it by dipole kicks  $\theta$  which are proportional to the offset, applied a quarter of the betatron oscillation downstream. We operate with dimensionless amplification factor  $g$  of the system (gain) which is equal to:

$$g = \frac{\theta \sqrt{\beta_1 \beta_2}}{X}, \quad (11)$$

where  $\beta_1$  and  $\beta_2$  are the beta-functions at the positions of the pick up and the kicker electrodes respectively. In the limit of  $g \ll 1$  the decrement due to the feedback is equal to  $\frac{1}{2} f_0 g$ , i.e. the amplitude of the betatron oscillations being reduced  $1/e$  times after  $2/g$  revolution periods. Theory of the feedback (see e.g. [5]) gives the transverse emittance evolution formula:

$$\frac{d\epsilon_n}{dt} = \left( \frac{4\pi \delta \nu_{rms}}{g} \right)^2 \left[ \left( \frac{d\epsilon_n}{dt} \right)_0 + \frac{\gamma f_0 g^2}{2\beta_1} X_{noise}^2 \right], \quad g \gg 4\pi \delta \nu_{rms}, \quad (12)$$

where emittance growth rate without feedback  $(d\epsilon_n/dt)_0$  is given by (1,4),  $X_{noise}$  is the rms noise of the system (presented as equivalent input noise at the pick-up position), and  $\delta \nu_{rms}$  is the rms tune spread within a beam.

**Sources of decoherence.** The decoherence of betatron oscillations is caused by several kinds of the tune spread [18, 19, 20]:

- rms tune spread due to nonlinear fields is about

$$\delta\nu_{NL,O} = \sigma^2(d\nu/da^2) \simeq \nu \frac{\epsilon_n \langle \beta \rangle^2}{\gamma} b_3 = 10^{-6},$$

due to systematic error octupole component of  $b_3 = 10^{-6} \text{ cm}^{-3}$  [3], and about twice larger due to sextupoles used for chromaticity correction  $b_2 \approx \nu / (\langle \beta \rangle \langle D_x \rangle) = 2.5 \cdot 10^{-4} \text{ cm}^{-2}$ :

$$\delta\nu_{NL,S} \simeq \frac{\epsilon_n \langle \beta \rangle^3}{2\gamma\nu} b_2^2 = \nu \frac{\epsilon_n \langle \beta \rangle}{2\gamma \langle D_x \rangle^2} \approx 2 \cdot 10^{-6},$$

- tune spread due to residual chromaticity and momentum spread

$$\delta\nu_{CR} \approx 2\nu_s \left( \frac{\eta(\Delta P/P)}{2\nu_s} \right)^2 \simeq 10^{-5}$$

if the chromaticity  $\eta$  is compensated down to 5, and the synchrotron tune is  $\nu_s = 2.4 \cdot 10^{-4}$ ;

- major source of the tune spread (and, consequently, decoherence) is nonlinear beam-beam force which results in the rms tune spread of [20]

$$\delta\nu_{BB} \approx 0.167\xi = 8.4 \cdot 10^{-4}.$$

The decoherence takes place over about  $N_{decoher} \approx 1/\delta\nu_{BB} \approx 1200$  turns.

**Ultimate gain and emittance growth reduction.** Computer simulations [4, 21] and analytical consideration of the feedback system [22] resulted in maximum useful gain factor  $g_{max} \simeq 0.3$  – there found no reduction of the emittance growth rate with further increase of  $g$  because of higher-(than dipole)-order kicks effect, the system noise contribution grows, while the coherent tune shift due to feedback becomes too large, and affects multibunch beam stability in presence of resistive wall impedance.

Therefore, maximum reduction factor  $R_{max} = (g_{max}/4\pi\Delta\nu_{BB})^2$  is about 800 for the Pipetron design parameter of  $\xi = 0.005$ , while the minimum practical gain which still can lead to the damping is about  $4\pi\delta\nu_{BB} \approx 0.01$ . Note, that DESY and SSC ground motion powers – see Table 2 – at  $f_1 = 0.18f_0$  are beyond the extreme feedback capability.

As it is seen from (12), feedback noise also leads to emittance growth and its relative contribution grows as  $\propto g^2$ . Taking the beta function at the pick-up  $\beta_1 = 500\text{m}$  we get limit on the rms noise amplitude:

$$X_{noise} \ll X_{noise}^{max} = \left[ \frac{2\beta_1 (d\epsilon_n/dt)_0}{f_0 (4\pi\delta\nu_{BB})^2 \gamma} \right]^{1/2} \approx 1.4 \mu m. \quad (13)$$

Thermal noise at room temperature  $T$  for a pick-up with half-aperture  $b$  can be estimated as:

$$X_{th} = \frac{b}{f_0 N_p N_b e} \sqrt{\frac{4kT\Delta f}{Z}} \approx 0.5[nm] \sqrt{\Delta f[kHz]}, \quad (14)$$

here  $k$  is Boltzmann constant; pick-up impedance was chosen  $Z = 50 \text{ Ohm}$ . For a narrow band system with  $\Delta f \sim 10 \text{ kHz}$ , the noise is about  $1.6nm$ , while for a bunch-by-bunch feedback system  $\Delta f = 10MHz$  and  $X_{th} = 0.05 \mu m$ . We see, that, in principle, thermal noise limit is well below the necessary accuracy of  $1.4 \mu m$  (see (13)).

Power of the output amplifier of the system depends on maximum noise amplitude of the proton beam oscillations. The rms coherent oscillation amplitude can be estimated as  $\delta X_{rms} \approx \sqrt{N_{decoher} N_q} B / f_1 \approx 2 \mu m$ . Taking the “safety “ factor of 5 we get  $\delta X_{max} = 5 \cdot \delta X_{rms} = 10 \mu m$  maximum amplitude, and the necessary angular kick of about  $2 \cdot 10^{-9} \text{ rad}$  – we assume  $\beta_2 = 500 \text{ m}$  at the kicker. Such a corrector with a length of  $l_k = 1m$ , and an aperture  $b = 1cm$  will require a certain amount of energy  $\delta W$  of electric (or magnetic) field  $E$ :

$$\delta W = \frac{E^2}{8\pi} \pi l_k b^2 \simeq \delta X_{max}^2 [\mu m] b^2 [cm] / l_k [cm] \cdot 5 [mJ] = 5 [mJ]. \quad (15)$$

Again, for a narrow band feedback system with  $\Delta f = 10 \text{ kHz}$ , it yields the power of  $P = \delta W \Delta f = 50 \text{ W}$ , while for a bunch-by-bunch system one needs  $50 \text{ kW}$  amplifier.

## 2.4 RF Phase Noise

Basic equation of the longitudinal particle motion describes particle motion under impact of the RF phase error  $\Delta\phi$ :

$$\begin{aligned} \left( \frac{\Delta p}{p} \right)_{n+1} &= \left( \frac{\Delta p}{p} \right)_n - \frac{eV_0}{E_p} \phi_n, \\ \phi_{n+1} &= \phi_n + 2\pi h \left( \frac{\Delta p}{p} \right)_n + \Delta\phi_n, \end{aligned} \quad (16)$$

here  $V_0$  stands for the RF voltage, harmonics number  $h = f_{RF}/f_0$ ,  $p$  is particle momentum. Turn-to-turn jitter of the RF phase results in fast momentum variation  $(\Delta p/p) = (eV_0/E_p)\Delta\phi$  which leads to an instant change of the horizontal orbit of  $\Delta X = D_x(\Delta p/p)$ , where  $D_x$  is the dispersion function at the RF cavities. It is

equivalent to beam displacement and – again, after decoherence process – causes the emittance growth of:

$$\frac{d\epsilon_n}{dt} = \frac{1}{2}\gamma H \delta\phi^2 f_0 \frac{eV_0}{E_p}, \quad (17)$$

where the invariant  $H = (D_x^2 + [\beta_x D'_x - \beta'_x D_x/2]^2)/\beta_x$ . The energy gain of 100 TeV over  $\tau_R = 0.5$  hour requires 185 MeV per turn energy increase, thus, taking an overvoltage factor of 2 we need  $eV_0 = 370$  MeV. Taking (in the worst case)  $H = 1$  cm at the RF system position, one gets that 10% emittance increase during the ramp time occurs with the rms turn-by-turn RF phase jitter  $\delta\phi \equiv \sqrt{f_0 \sum_n S_\phi(f_0|\nu - n|)} \simeq 5$  mrad. Note, that frequencies of interest are still of about  $f_1$  and  $f_0$ , i.e. of the order of hundred(s) of Hz. The measured one phase noise at the Tevatron is less than 0.04 in 100 Hz frequency band [23], i.e. more than 100 times less than the tolerance. There is no need of high voltage RF at the collision energy at the Pipetron, and, say,  $eV_0 = 20$  MeV should be enough, that yields in easier tolerances on the phase stability of  $\delta\phi \simeq 30$  mrad. Thus, the RF phase jitter does not seem to be a real problem for the transverse emittance degradation.

As it is seen from (16), fast variation of the voltage  $\delta V$  also can initiate the effect, and the tolerance on the amplitude can be derived from the phase tolerance as  $(\Delta V/V_0) \approx \Delta\phi_s \simeq 0.03$ , where  $\phi_s = \sigma_s/\lambda_{RF} \approx 0.15$ . This requirement also seems to be quite easy to fulfil.

### 3 Longitudinal Emittance Growth

#### 3.1 RF Noise Effect

The RF phase errors at frequencies of the order of synchrotron one  $f_s = \nu_s f_0$  and higher lead to the longitudinal emittance growth of:

$$\frac{dA}{dt} = \frac{eV_0}{f_{RF}} \frac{d\phi^2}{dt}. \quad (18)$$

The synchrotron oscillations phase grows under impact of noise as

$$\frac{d\phi^2}{dt} = \pi\omega_s^2 S_\phi(\omega_s) = 2\pi f_0^2 \nu_s^2 S_\phi(f_0 \nu_s)$$

, where  $\omega_s = 2\pi\nu_s f_0 > 0$ ,  $S_\phi$  is the PSD of the phase noise <sup>6</sup> (see e.g. Appendix C in [21]).

The synchrotron frequency

$$f_0 \nu_s = f_0 \sqrt{\alpha h e V_0 / (2\pi E_p)} = 0.017 [Hz] \sqrt{V_0 [MV] / (E_p / 100 TeV)}$$

---

<sup>6</sup>here the PSD in  $\omega = 2\pi f$  domain relates to  $f$  domain PSD as  $S(\omega) = S(f)/(2\pi)$ . Extended analytical consideration of the longitudinal emittance growth can be found in e.g. [24, 25].

varies from 3.1 Hz at the beginning of the ramp <sup>7</sup> ( $E_p=2$  TeV,  $V_0 = 370$  MV,  $\nu_s \approx 0.01$ ) to 0.33 Hz at the end of the ramp at 100 TeV ( $\nu_s \approx 0.0011$ ), and then it is about 0.076 Hz during the collision time with  $V_0 = 20$  MeV. The latter frequency corresponds to the synchrotron tune of  $\nu_s = 2.5 \cdot 10^{-4}$  which comes from single bunch stability threshold of the transverse mode-coupling instability:

$$\nu_s = \frac{16\sqrt{\pi}(E_p/e)\sigma_s}{2I_s R \text{Im} < Z_{\perp} \beta >}, \quad (19)$$

where  $I_s = 2 \mu\text{A}$  is DC single bunch current, and transverse impedance comes mostly from resistive walls  $\text{Im}Z_{\perp} = 377\Omega(R\delta/b^3) \simeq 240 M\Omega/\text{m}$  (the skin depth  $\delta$  for 10-cm long bunch in Al chamber is about  $4 \mu\text{m}$ ).

If one requires less than 10% emittance increase during half an hour of ramp time  $\tau_R$ , than the tolerance on the phase jitter PSD in  $f_{RF} = 450$  MHz RF system is:

$$S_{\phi}(\omega_s) = \frac{0.1 A f_{RF}}{\tau_R (e V_0) \pi \omega_s^2} \approx \frac{6.4 \cdot 10^{-6}}{\omega_s^2}. \quad (20)$$

Measurements with the SSC RF system HP8662 synthesizer [24] shows that in frequency band of 1-100 Hz the PSD of phase noise can be approximated by

$$S_{\phi}(\omega_s) = \frac{1.3 \cdot 10^{-5}}{\omega^{2.65}}, \quad (21)$$

that is twice the tolerance (20) at frequencies about 1 Hz.

Equivalent rms phase jitter tolerance is  $\delta\phi \simeq \sqrt{\omega_s S_{\phi}(\omega_s)} \approx 0.3$  mrad at  $f_s = 3$  Hz.

The same 10% tolerance for 5 hours of the collision operation with  $e V_0 = 20$  MeV gives:

$$S_{\phi}(\omega_s) \approx \frac{1.2 \cdot 10^{-5}}{\omega_s^2}. \quad (22)$$

that is very close to the measured PSD.

Having these numbers one can conclude that with some improvement of the RF phase stability with respect to the SSC synthesizer, no longitudinal feedback will probably be required. If the feedback will be implemented it should be not so sophisticated as transverse one – it should not be fast and have a large gain, because the process of the synchrotron oscillations decoherence takes hundreds of thousands of turns in the Pipetron. Tolerance on the RF voltage stability  $\delta V$  also does not seem tough – it can be estimated as  $(\delta V/V_0) \sim (\delta\phi/\phi_s) \simeq 0.2\%$  where we take acceptable phase jitter of 0.3 mrad, and the bunch phase area of  $\phi_s = \sigma_s f_{RF}/c \approx 150$  mrad.

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<sup>7</sup>here we take the momentum compaction factor of  $\alpha \approx 1/\nu_x^2 \approx 4 \cdot 10^{-6}$



### 3.2 Transverse Kicks Effect

Another possible source of the RF phase errors is the change of the circumference due to non-zero dispersion function  $D_x$  at the position of dipole kick [25], produced e.g. by displaced quadrupole magnet  $\theta = \Delta X/F$ :

$$\Delta\phi = 2\pi h D_x \theta = 2\pi h D_x \Delta X/F.$$

For the whole ring of  $N_q$  quadrupoles randomly moving at frequencies about  $f_s$  with rms amplitude of  $\delta X$ , it results in rms phase error:

$$\delta\phi = \frac{h \langle D_x \rangle \sqrt{N_q} \delta X}{F R} \approx \frac{h \sqrt{N_q} \delta X}{\nu_x^2 F}. \quad (23)$$

Combining (23) and (20), and taking  $h = 1.5 \cdot 10^6$ ,  $\nu_x \approx 500$ ,  $F \simeq 200$  m and  $N_q = 4000$  we get the tolerable PSD of ground motion <sup>8</sup>:

$$S_X(f_s = \nu_s f_0) = \frac{2.8 \cdot 10^5}{f_s^2} [\mu m^2 / Hz],$$

or about 300  $\mu m$  rms amplitude in 3 Hz frequency band.

As it is seen from Fig.1, the power of the ground noise at all probable synchrotron frequencies of 0.7–3 Hz is some 10000 times smaller, therefore the quadrupole motion effect is negligible. <sup>9</sup>

Quite similar consideration of the dipole field variation effect results in tolerance on the field stability of about  $(\delta B/B) \simeq 0.1\%$  rms in 3 Hz frequency band. Unfortunately, we have no available experimental data on the field stability, but the tolerance we got should not be severe.

## 4 Closed Orbit Distortions

### 4.1 Alignment Tolerances

The rms closed orbit distortion  $dX_{COD}$  is proportional to the rms error  $dX$  of quads alignment, and if these errors are not correlated, then in the FODO lattice we can get:

$$dX_{COD}^2 = \frac{\beta dX^2}{4 \sin^2(\pi \nu)} \sum_i \frac{\beta_i}{F_i^2} = \frac{\beta N_q t g(\mu/2) dX^2}{L \sin^2(\pi \nu)}. \quad (24)$$

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<sup>8</sup>in  $f$  domain

<sup>9</sup>the PSDs in Fig.1 are for *absolute* movements, i.e. those measured at one point by use of velocimeter seismic probe with further integration. Relative displacement is even smaller – see next Section on ground drifts.

Let us take the “safety criteria”, i.e. ratio of maximum allowable COD to the rms one, equal to  $5^{10}$ , then for maximum COD of  $dX_{COD}^{max}=1$  cm (this is about half aperture of the vacuum chamber) at the focusing lenses where  $\beta_F = 765$  m ( $L = 250$  m,  $\mu = 90^\circ$ ) we get requirement on the rms alignment error of  $dX \approx 15$   $\mu$ m (there was used the value of tune  $\Delta\nu = 0.31$ ). This value sets a challenging task, its solution needs the most sophisticated alignment techniques and two questions arise in this connection: 1) temporal stability of the magnets positions; and 2) applicability of the beam-based alignment.

## 4.2 Slow Ground Motion

Numerous data on uncorrelated slow ground motion support an idea of “space-time ground diffusion”. An empirical rule that describes the diffusion – so called “the ATL law” [26] – states the rms of relative displacement  $dX$  (in any direction) of two points located at a distance  $L$  grows with time interval  $T$ :

$$\langle dX^2 \rangle = ATL, \quad (25)$$

where  $A$  is site dependent coefficient of the order of  $10^{-5\pm1}$   $\mu m^2/(s \cdot m)$ . As long as the diffusion coefficient  $A$  is very small, the ground wandering presents only a tiny, but important contribution to the total ground motion which can be several orders of magnitude larger but well correlated in space and time at very low frequencies, systematic, unidirectional, and, therefore, sometimes predictable. The PSD of ATL diffusion is equal to

$$S_{ATL}(f) = AL/(2\pi^2 f^2). \quad (26)$$

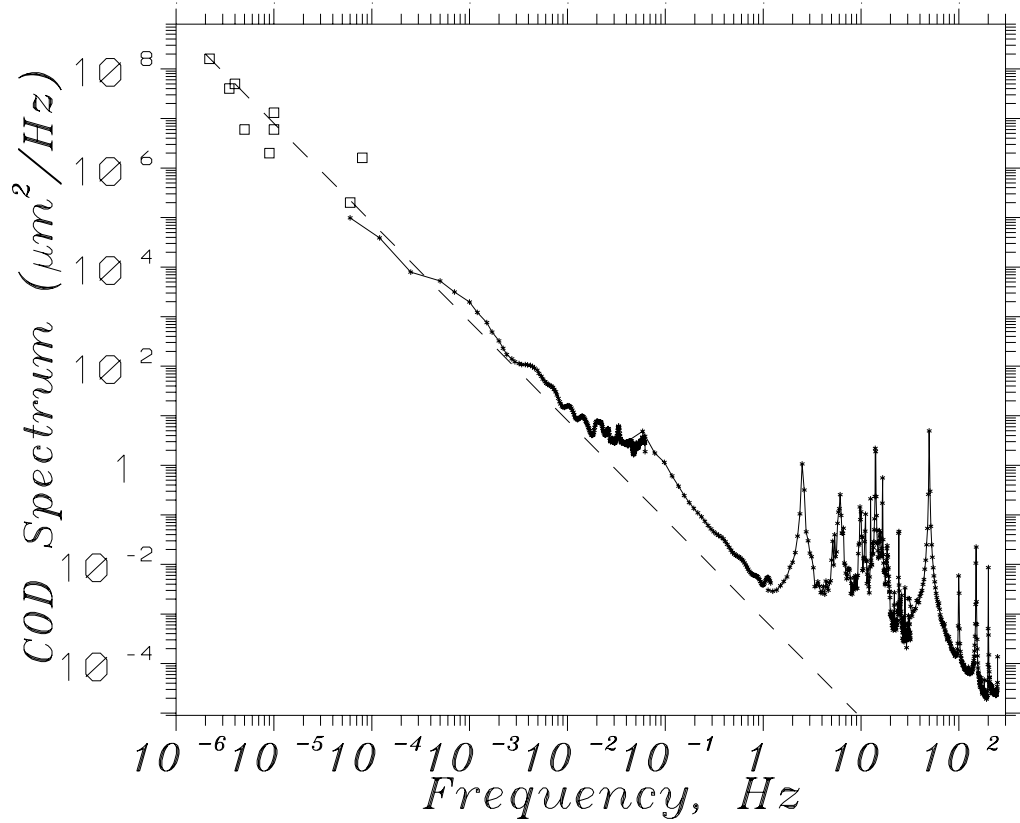
The ground diffusion should cause corresponding COD diffusion in accelerators with rms value equal to [27]:

$$\langle dX_{COD}^2 \rangle = \frac{\beta AT C (\beta_F + \beta_D)}{8F_0^2 \sin^2(\pi\nu)}, \quad (27)$$

here  $C$  is the accelerator circumference,  $F_0$  is the focal length of each quadrupole in  $FODO$  lattice,  $\nu$  is the tune of the machine,  $\beta$  is the beta-function at the point of observation. For most of practical estimations of the rms orbit distortion amplitude averaged over the ring, the formula  $COD \simeq 2\sqrt{ATC}$  can be used. It clearly shows that the diffusive orbit drift is not very sensitive to the focusing lattice type (only the circumference  $C$  plays role), in particular, there is almost no difference between the combined- and separated-function lattices responses on the  $ATL$ -like diffusion.

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<sup>10</sup>Let us remark that probably this factor of 5 will not be enough in the Pipetron with its challenging tolerances, because recent accelerator alignment studies at SLAC and Japan [28, 29] show that due to both human and natural factors, the alignment errors statistics is far from Gaussian, it is rather power-law-like, it often has no finite variance value and demonstrates significant probability to have many-sigma outliers.



**Figure 4:** Spectrum of vertical orbit drifts at HERA-p normalized on  $\beta = 1\text{m}$ .  
Dashed line is for the *ATL* model prediction.

Fig.4 presents the PSD of the HERA- $p$  vertical orbit (scaled for  $\beta = 1\text{ m}$ ) which clearly demonstrates “diffusion-like” behavior of the COD at frequencies below 0.1 Hz – the dashed line is for  $S_{COD}(f) = 8 \cdot 10^{-4}/f^2 [\mu\text{m}^2/\text{Hz}]$  which is in agreement with the *ATL* law with  $A = 3.8 \cdot 10^{-5} \mu\text{m}^2/(s \cdot \text{m})$  (see formula (26) above). Peaks above 2 Hz are due to technological equipment. The squares at lower frequencies represent the Fourier spectra of proton orbit in 131 BPMs from different fills of the storage ring [30]. Solid line is for data from a low noise BPM [9]. The motion of quads was checked to be the only candidate that can explain these drifts. It was stressed in [30], that having completely different magnet lattice, the HERA electron ring orbit also performs “random-walk-like” diffusion with comparable coefficient  $A$ .

Review of the ground diffusion observations [31] points out that the diffusion coefficient  $A$  depends on tunnel depth and type of rock.<sup>11</sup> The question of the limits

<sup>11</sup>Linear Collider study group at KEK reported indication of significant (15 times in the coefficient  $A$ ) seasonal variations of the diffusion in the 300-m-deep Sazare mine (Japan, green schist) [32] and they also observed 5 time larger  $A$  in a dynamite-dug tunnel in welded tuff with respect to drilled tunnel in granite (i.e. the tunnel construction method probably makes a difference) [33].

of applicability of the  $ATL$  law is still open – available data cover  $T$  from minutes to dozen years,  $L$  from meters to dozens km.

Let us scale the HERA- $p$  orbit data from Fig.4 to the Pipetron with use of Eq.(27) (i.e one should replace  $\beta_F + \beta_D$  from 94.2 m at HERA to 1000 m at the Pipetron,  $C$  from 6.3 km to 1000 km,  $F_0$  from 16.8 m to 177 m, and  $\Delta\nu$  from 0.298 to 0.31) then we obtain rms COD at  $\beta_{max} = 850$  m equal to:

$$dX_{COD} \approx 800[\mu m] \sqrt{T[hrs]}. \quad (28)$$

Again, requiring “safe” rms COD of 2 mm, we get  $T=6.3$  hours mean time between necessary realignments to initial “smooth” orbit.

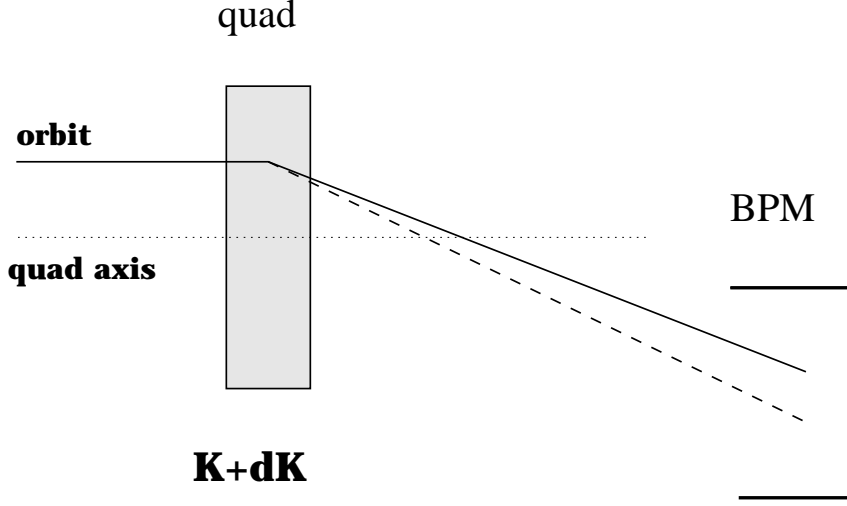
If one intends to have a stable and deep tunnel comparable with the LEP one where it was found  $A \approx 5 \cdot 10^{-6} \mu m^2/(s \cdot m)$ , then the corresponding orbit drift is  $dX_{COD} \approx 800[\mu m] \sqrt{T[hrs]}$  and the period of necessary repetition of the Pipetron alignments is about 2 days. It does not seem to be an easy task to do it mechanically, even with use of robots, especially taking into account 15  $\mu m$  precision of the procedure. “Beam-based alignment” technique looks as the most appropriate for that.

### 4.3 Correction System

“Beam-based alignment” assumes an extensive use of BPM readings in order to utilize information about beam distortions for the “golden” orbit maintenance. In circular accelerators this method (also named “K-modulation”) is based on a fact that if the strength of a single quadrupole  $K = Gl/Pc$  in the ring is changed by  $dK$ , the resulted difference in closed orbit is proportional to the original offset of the beam in the quadrupole – see Fig.5.

From the measured difference orbit the offset can be determined, yielding either the quad offset to eliminate or the offset between quadrupole axis and BPM adjacent to the quad for global correction. The method is widely used now at many accelerators, e.g. in HERA-e all of 148 quads were equipped with switches in order to vary the strength of magnets individually, that allows to align the ring within 0.05 mm error in less than 24 hours [34].

For the Pipetron, the tolerance on quads alignment of  $dX = 15 \mu m$  yields in beam displacement in the next downstream quadrupole position (where we assume the BPM) of the order of  $dXL/F(dK/K) \simeq 1 \mu m$  if the modulation depth is about  $dK/K = 0.05$ . Taking several measurements or/and with use of phase-lock technique one can distinguish such displacement with BPM resolution of the order of  $\Delta_{BPM} \simeq 5 \mu m$ .



**Figure 5:** Principle of the beam-based alignment.

Let us calculate necessary strength of correctors assuming two correctors per cell, geologically stable tunnel (deep, in the hard rock) which can be characterized by the ground diffusion coefficient  $A = 5 \cdot 10^{-6} \mu m^2 / m / s$  (close to LEP tunnel data [31]) and requiring that no mechanical realignment will be necessary within  $T=10$  years period. Accordingly to the ATL law (25) it gives  $\sqrt{ATL} \approx 630 \mu m$  rms relative quads displacement ( $L = 250m$ ), or (factor of 5) about  $dX_{max} = 3.2 mm$  of maximum displacement. Thus, the maximum angle to correct is  $dX_{max}/L \simeq 13 \mu rad$ , or about 4.3 Tm of the corrector strength at 100 TeV.

## 5 Discussion

Table 3 compares tolerances for hadron colliders of LHC(CERN), SSC and the Pipe-tron. There are two major effects which limit collider performance. The first is the transverse emittance growth due to fast (turn-to-turn) dipole angular kicks  $\delta\theta$  produced by bending field fluctuations in dipole magnets  $\Delta B/B$  or by fast motion of quadrupoles  $\sigma_q$ . The 10% emittance increase requirement  $d\epsilon_n/dt < 0.1\epsilon_n/\tau_C$ , where  $\tau_C$  is the collision regime duration, sets a limit on the turn-by-turn jitter amplitude which looks extremely tough – of the order of the atomic size! Comparison with results of measurements shows that for all three colliders the effect may have severe consequences, although the Pipetron is the most troublesome case.

Other figures in Table 3 are for the rms quad-to-quad alignment tolerances in order to keep the rms orbit  $dX_{COD}$  within 5 mm, and the estimated time after which cumulative drifts due to ground diffusion will cause these distortions  $T_c \approx dX_{COD}^2 / (4AC)$  (we take here  $A = 10^{-5} \mu m^2 / (s \cdot m)$ ). One can see that the SSC and the Pipetron

have to be realigned very often – or, another solution, to have strong and numerous correctors.

Table 3: Stability of Hadron Colliders

Parameter	LHC	SSC	Pipetron
Energy $E$ , TeV	7	20	100
Circumference $C$ km	26.7	87.1	1000
Emittance $\epsilon_n$ , $\mu\text{m}$	4	1	1
$L$ -lifetime $\tau_C$ , hrs	10	20	5
$\Delta\nu f_0$ , Hz	3100	760	54-135
Quads jitter $\sigma_q$ , nm	0.05	0.03	0.008
Measured jitter, nm	0.01-0.1	0.2	0.1-50
$\Delta B/B$ , $10^{-10}$	$\sim 4$	$\sim 2$	$\approx 3.4$
Align. error, $\mu\text{m}$	100	60	40
Realign. time, $T_c$	$\sim 1.5$ yr.	$\sim 6$ mos.	$\sim 2$ weeks

Preceding consideration has shown that natural and man-made vibrations at Pipetron can lead to dangerous transverse emittance growth rate (high-frequency part of spectrum) and closed orbit distortions (at lower frequencies). At the early stage of the project, “on-site” ground motion measurements are necessary to conclude

- 1) are the measured vibrations dangerous for the Pipetron beam dynamics?
- 2) (if - presumably - yes) what are necessary parameters of the beam emittance preservation feedback system (gain, noise, bandwidth, power) and strength of dipole orbit correctors?

For that it seems reasonable to investigate experimentally following topics:

- amplitudes of vibrations, their spectra in 0.01–300 Hz band,
- correlation of vibrations at distances of 0...500 m,
- amplitudes in a tunnel (Tevatron or test tunnel) vs. surface ones,
- influence of weather (thunderstorm, wind, rain, temperature changes),
- ground motion at FNAL and at other probable site(s),
- influence of traffic, other high frequency cultural noise,
- impact of quarry blasts, remote and local earthquakes,
- mechanical resonances of the magnet prototype,
- emittance growth modeling with seismometers “on-line” (as in [35]),

- relative drifts of tunnel floor over long periods of time (days–months) at distances from dozen meters to a kilometer.

Besides these items, the Pipetron emittance growth rate estimations call for measurements of:

- the RF system phase and amplitude noises in frequency band of 0.01–500 Hz,
- periodical ripple and random noise in magnitude of dipole magnetic field in 0.01–500 Hz band,
- spatial correlation of the bending magnetic field jitter along 250-m long dipole magnet.

## 6 Conclusions

In this article we have studied impact of external noises on the Pipetron proton collider transverse and longitudinal beam dynamics. General conclusion is that there are several rather tough requirements on the noise amplitudes but they can be fulfilled.

In more detail, we found that:

Acceptable transverse emittance growth rate (less than 10% over the beam lifetime) requires less than 0.076 nm turn-to-turn uncorrelated jitter of the quadrupole positions and less than  $3.4 \cdot 10^{-10}$  field strength fluctuations in dipole magnets. Analysis of up-to-date ground motion measurements worldwide shows that these tolerances are too tight for actual accelerator tunnels. The emittance growth due to ground motion is smaller for larger fractional part of the betatron tune, and we suggest to have  $\Delta\nu$  (or  $1 - \Delta\nu$ ) as big as 0.3-0.45. There is a certain need in a feedback system to damp betatron oscillations and reduce the growth. Decoherence due to beam-beam interaction in the Pipetron is too fast, and limits the maximum transverse emittance growth rate reduction factor by the value of about 800. We also found that thermal noise in the feedback BPM will not limit the system performance, and estimated necessary power of system with the 10 MHz frequency band to be about 50 kW. It is noted that combined function magnetic structure of the collider is preferable as it eases the tolerances.

Estimates based on the Tevatron and the SSC RF systems phase errors measurements, show that the RF phase jitter in Pipetron will not cause any significant transverse emittance growths, while only several-fold improvement in the phase stabilization at low frequencies will allow to avoid longitudinal feedback system as well. Low frequency quadrupole movements will not cause the bunch lengthening due to synchrotron coupling with non-zero dispersion in the ring.

Maximum distortions of the proton closed orbit of the order of the vacuum chamber size were found to occur with some  $15\text{ }\mu\text{m}$  rms relative quad to quad misalignment which is – accordingly to the HERA- $p$  observations and the “ATL law” – to be accumulated during 6 hours of operation. To counteract the effect the beam-based alignment technique must be implemented, that requires some  $5\text{ }\mu\text{m}$  BPM accuracy, and 4.5 Tm corrector strength, but in return will allow to avoid mechanical realignment with use of robots over 10 years time periods.

Finally, we emphasize an importance of “on-site” ground motion studies and magnet vibrations measurements, as well as necessity of data on long-term tunnel movements, the RF phase and amplitude stability, and dipole field jitter.

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